

عدد المسائل: ثلاث	مسابقة في مادة الرياضيات المدة: ٧٥ دقيقة	الاسم: الرقم:
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إرشادات عامة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الإلتزام بترتيب المسائل الواردة في المسابقة.

I- (6 points)

The table below shows the annual average price (in dollars) of one ounce of gold (x_i) and one ounce of silver (y_i) over five consecutive years.

Year	2019	2020	2021	2022	2023
Annual average price of one ounce of gold (in dollar): x_i	1799	1801	1943	2387	3295
Annual average price of one ounce of silver (in dollar): y_i	24.14	25.73	m	28.27	32.8

The plane is referred to an orthogonal system. All values are rounded to the nearest 10^{-4} .

Part A

- 1) The correlation coefficient of this statistical data (x_i, y_i) is $r = 0.9822$. Interpret this result.
- 2) The coordinates $\bar{x}$ and $\bar{y}$ of G, the center of gravity of this statistical data (x_i, y_i) are:
 $\bar{x} = 2245$ and $\bar{y} = 27.458$.
 - a) Calculate m.
 - b) The equation of the regression line ($D_{y/x}$), of y in terms of x, is given by
 $y = 0.0052x + b$ where b is a real number. Show that $b = 15.784$.

Part B

Suppose that the preceding model remains valid till the year 2030.

- 1) In 2024, the annual average price of one ounce of gold was 4000 dollars.
Estimate the annual average price of one ounce of silver.
- 2) Assume that during a certain year before 2030,
the annual average price x of one ounce gold increases by 10%.
 - a) Determine, in terms of x, the corresponding increase in the annual average price of one ounce of silver.
 - b) Suppose that the percentage increase in the annual average price of one ounce silver is 5.6, estimate the annual average price x of one ounce gold.

II- (5 points)

In the table below, only one among the proposed answers to each question is correct.

Write the number of each question and give, with justification, the answer that corresponds to it.

N°	Questions	Proposed answers		
		a	b	c
1	For all real number $x \neq -\frac{\ln 3}{3}$, $\frac{\ln(8e^{2x}e^{4x} + (e^{3x})^2)}{3x + \ln 3} =$	2	$\frac{54x}{3x + \ln 3}$	$\frac{9 + 6x}{3x + \ln 3}$
2	The number of solutions, in $\mathbb{R}$, of the equation $(e^x - 1)(\ln(x - 1)) = 0$ is	0	1	2
3	$\lim_{x \rightarrow +\infty} \frac{x}{\ln(5x - 7)} =$	0	1	$+\infty$
4	The set of solutions, in $\mathbb{R}$, of the inequality $\ln(-x + 7) \leq \ln 5$ is:	$[2, +\infty[$	$[2, 7[$	$] -\infty, 2[$
5	Consider the function f defined as $f(x) = \begin{cases} e^{x-2a} & \text{for } x \leq 1 \\ 1 + \ln x & \text{for } x > 1 \end{cases}$ If f is continuous then a =	0	$\frac{1}{2}$	1

III- (9 points)

The plane is referred to an orthonormal system $(O ; \vec{i}, \vec{j})$.

Consider the function f defined on $\mathbb{R}$ as $f(x) = 1 - 2x - 2e^{-x-1}$ and denote by (C) its representative curve.

Let (d) be the line with equation $y = 1 - 2x$.

1) Show that $\lim_{x \rightarrow -\infty} f(x) = -\infty$ and calculate $f(-3)$.

2) a) Determine $\lim_{x \rightarrow +\infty} f(x)$.

b) Prove that (C) is below (d) for all $x \in \mathbb{R}$.

c) Prove that (d) is an asymptote to (C) .

3) a) Calculate $f'(x)$.

b) Copy and complete the table of variations of f .

x	$-\infty$	-1	$+\infty$
$f'(x)$		0	
$f(x)$			

c) Deduce that $f(x) \leq 1$ for all $x \in \mathbb{R}$.

4) Show that the equation $f(x) = 0$ has two distinct solutions α and β .

5) Knowing that $-1.87 < \alpha < -1.86$ and $0.19 < \beta < 0.21$.

Draw (d) and (C) .

6) Let h be the function given by $h(x) = \ln(1 - f(x))$.

a) Determine the domain of definition of h .

b) Do the two functions f and h have the same sense of variations over $]-\infty, -1[$?

Justify your answer.